



Channel Islands
CALIFORNIA STATE UNIVERSITY

Brian Rasnow, Timo Schulze, Ravneet Singh, Brian J Clark

Introduction

The Arctic will be free of ice within decades or sooner (Hansen et al. 2016). The temperature profile of a melting ice cube provides a readily understandable and compelling context to discuss implications of this climate change and meta-issues, such as the significance of climate models, data, and the scientific method. We describe a very simple, inexpensive, and accurate apparatus to automatically measure and log the temperature of melting ice in a classroom. We show how adding random temporal noise to the automated measurements can *improve* their precision. We finally present observations and results of the demonstration in physics, chemistry, and University (general ed) classes.

The classroom activity begins by asking students to predict the temperature evolution, $T(t)$, of a small sample of H_2O upon removal from a -80°C freezer (Fig. 1).

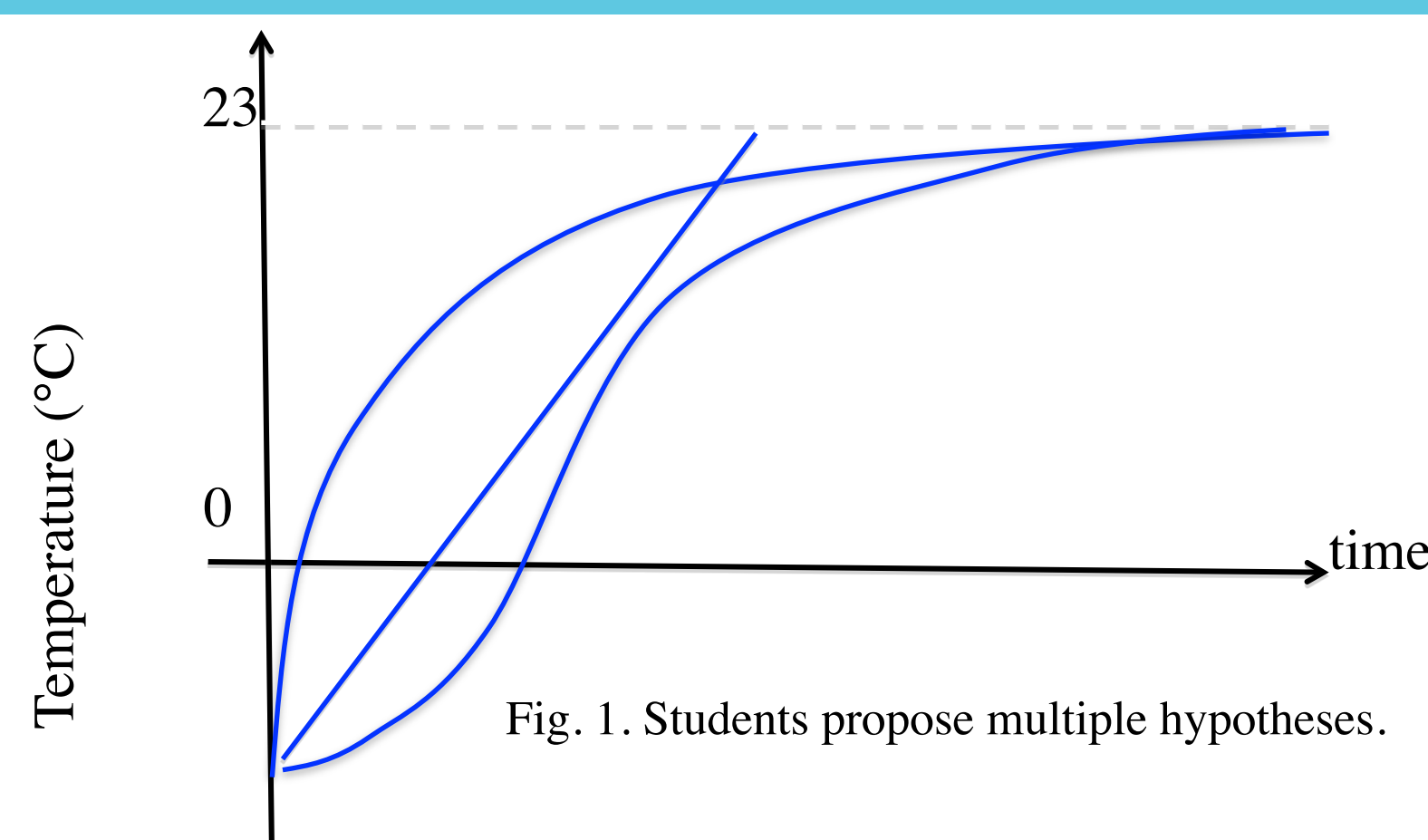


Fig. 1. Students propose multiple hypotheses.



Methods

The temperature of the ice was measured with a thermocouple frozen in ~ 1 cc of water in a test tube. First the thermocouple was dipped into latex paint to provide thin electrical insulation. For initial trials, the temperature was read with a digital multimeter and logged manually, using a cell phone for timekeeping. An automated solution using a MAX6675 thermocouple-to-digital converter evaluation kit (<http://datasheets.maximintegrated.com/en/ds/MAX6675.pdf>) and an Arduino Uno microcontroller (www.arduino.cc) has advantages of more frequent and potentially more accurate measurements at lower cost, and ease of moving the data into numerical software such as Excel or Matlab (Fig. 2).

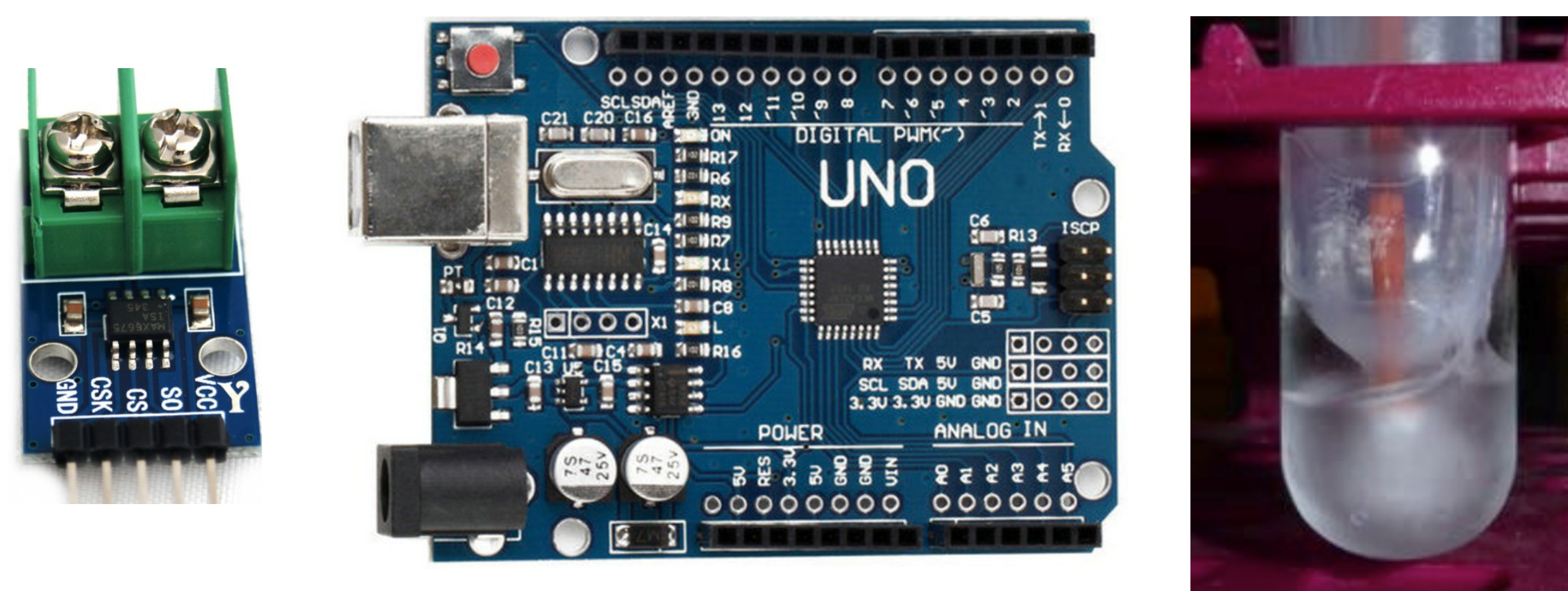


Fig. 2. MAX6675 thermocouple interface (\$3), Arduino (\$4), and thermocouple (\$2).

Initial Results

The MAX6675 has 12 bit, $\frac{1}{4}^\circ\text{C}$ resolution, but exhibited much greater noise, with standard deviation $\sim 1^\circ\text{C}$ (Fig. 3). Averaging N measurements was expected to reduce the noise by \sqrt{N} , but failed to do so suggesting the noise is not independent. We soon recognized that if queried multiple times within the ~ 200 msec conversion time, the MAX6675 will return the previous value, but this wasn't the only violation of statistical independence in the noise.

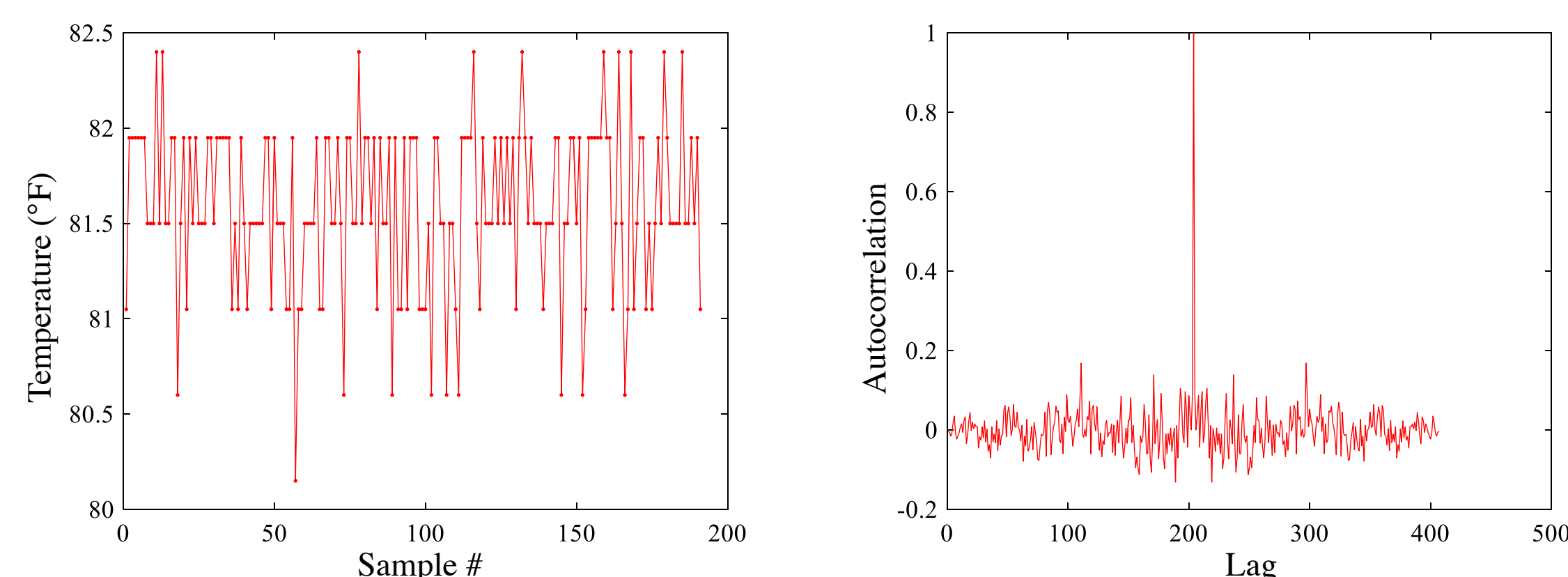


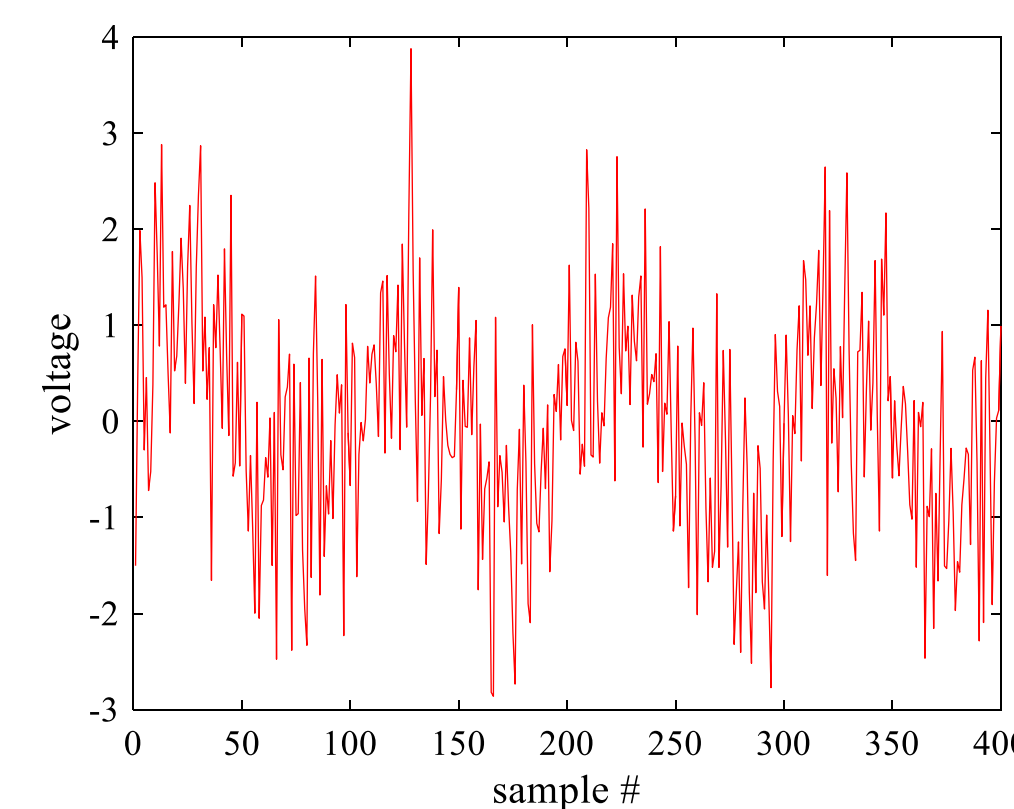
Fig. 3. A. 198 consecutive temperature measurements taken ~ 200 msec apart show standard deviation exceeding the digitization noise. B. Autocorrelation of ~ 206 averages of 10 consecutive samples suggests the noise is primarily Markovian.

Noise Simulations

Using Matlab, we simulated recovering a slow signal (zero in the simulation) in the presence of both random noise and 60 Hz electrical noise. Averaging in phase with 60 Hz or over a non-integer fraction of a 60 Hz period could result in sample bias, as demonstrated by the following code:

```
noise = randn(1e5,1);
% 1000 periods of 60 Hz
sixty = sin(2*pi * [1:1e5]' / 100);
data = noise + sixty;
plot(data(1:400));
xlabel('sample #'); ylabel('voltage')

std([noise sixty data])
ans =
    0.9985    0.7071    1.2222
```



The standard deviation of the noise is 1 by construction. The sine wave has rms amplitude $1/\sqrt{2}$. Added together in quadrature should yield a standard deviation of

```
>> sqrt(1^2+.707^2)
ans =
    1.2247
```

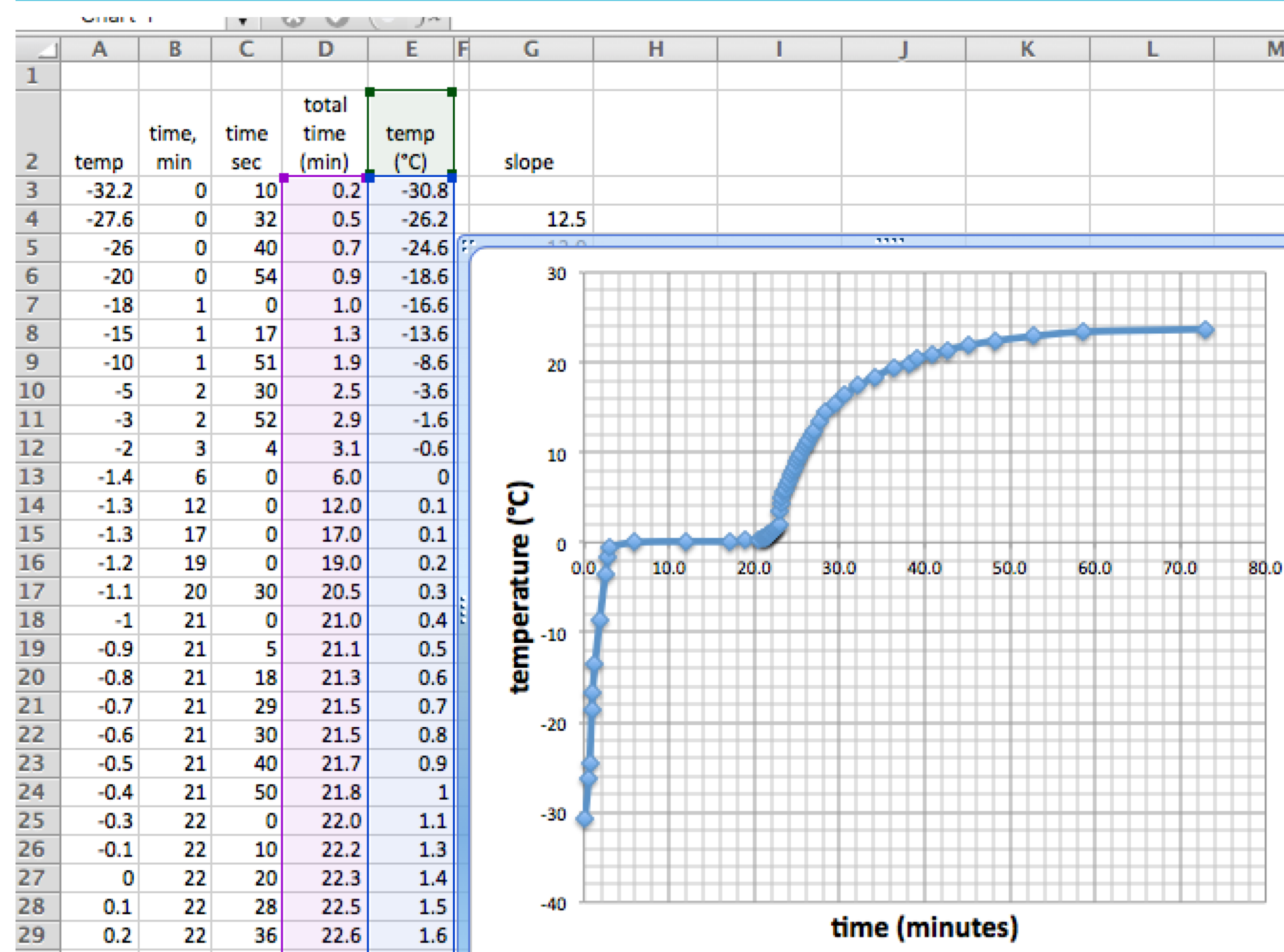
If we average over $\frac{1}{2}$ and 1 cycle of 60 Hz, and also over 50 samples chosen at random phase:

```
m50 = mean(reshape(data, 50, 1e5/50)); % average over 1/2 cycle
m100 = mean(reshape(data, 100, 1e5/100)); % average over 1 60 Hz cycle
% average over 50 random phases
d = reshape(data, 100, 1e5/100);
for i=1:1000, r50(i) = mean(d(randperm(100,50),i)); end
[std(data) std(m50) std(m100) std(r50)]
ans =
    1.2222    0.6494    0.1003    0.1554
```

If the noise and 60Hz were random, the 50 point average would have a standard deviation $\sim 1.222/\sqrt{50} = 0.173$ and the 100 point average would have a standard deviation of $1.222/\sqrt{100} = 0.122$. But because the 60Hz contamination is periodic, the 50 point average over alternating positive and negative halves of its period adds significant variance. Conversely, averaging over integer periods completely eliminates the 60Hz component, rather than attenuating it by \sqrt{N} . If averaging over an integral number of periods is difficult because of timing uncertainty, then adding random temporal noise between samples (e.g., `delayMicroseconds(random(1E6/60));`) provides an intermediate benefit, in this case reducing the standard deviation by a factor of $1.222/0.1554 = 7.86$ – slightly *more* than $\sqrt{50}$.

Melting Ice Kinetics

The following temperature data was collected manually and plotted in Excel.



Discussion & Metacognition

Most students (except some chemists) are surprised at the resulting curve. Do we believe it? We explore possible sources of errors, e.g., there appears to be a systematic error of $\sim 1.4^\circ\text{C}$, but ultimately we rationalize the curve is very consistent with physical properties of water and ice (Table), and our intuitions are wrong. We construct a model that succinctly explains the data.

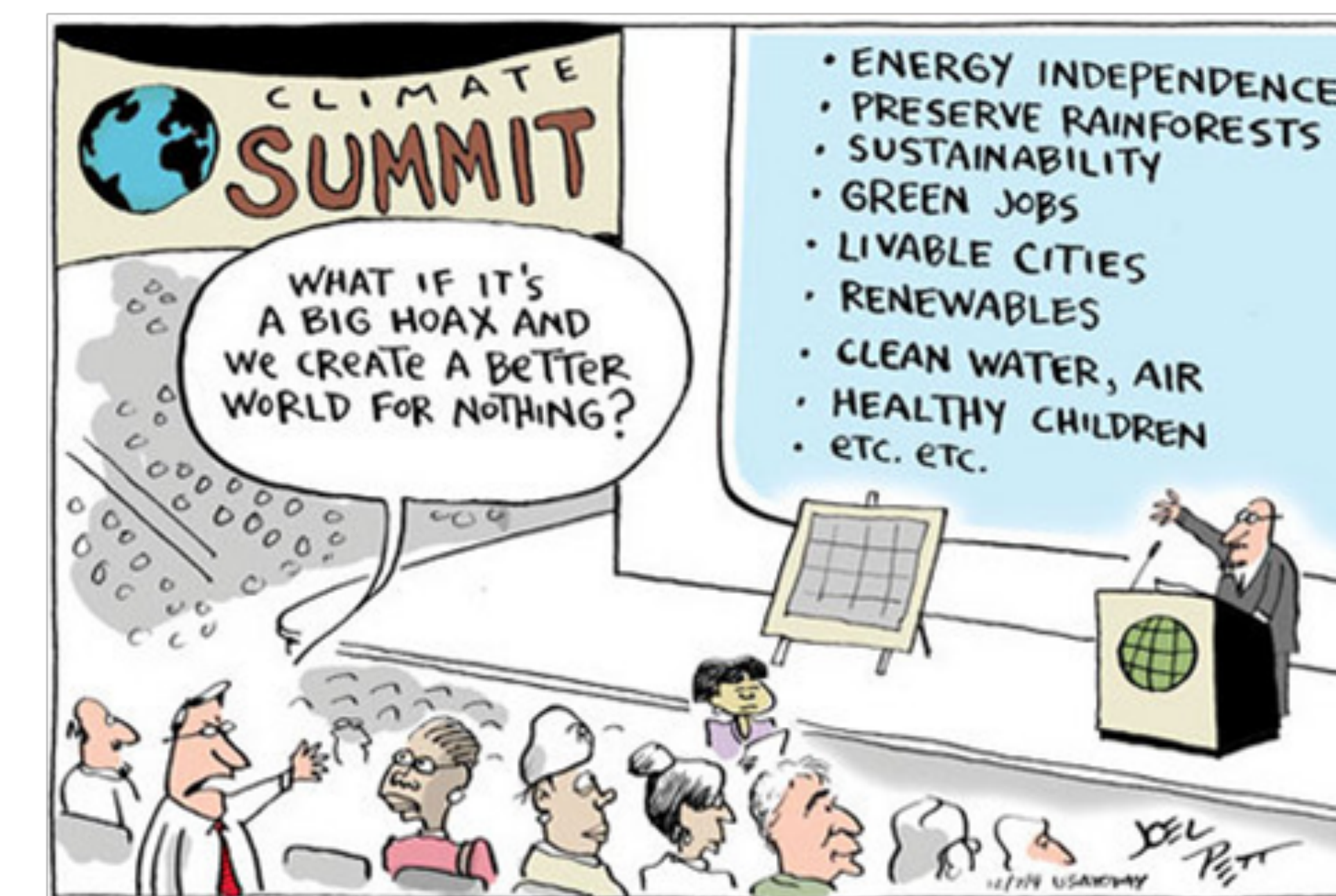
Our model: Heat energy flows from the warm room to the ice, raising its temperature through its $2.1 \text{ J/g}^\circ\text{C}$ specific heat. At 0°C the inflowing energy melts the ice isothermally through its 330 J/g (huge) latent heat. After all the ice has melted, the temperature takes off again via the $4.2 \text{ J/g}^\circ\text{C}$ specific heat of water. This model explains the general curve as well as subtleties such as the ratio of the slopes on either side of the plateau.

Property	Value
Latent heat of melting	330 J/g
Specific heat of water	4.2 J/gK
Specific heat of ice	2.1 J/gK
Thermal conductivity of water	0.6 W/mK
Thermal conductivity of ice	2.2 W/mK

Implications for Global Warming

What does the scientific model of ice melting summarized here imply about global warming in the Arctic? Where on our temperature curve is the Arctic now? Although the Arctic is obviously more heterogeneous than a test tube, ice absorbs 330 J/g up there as well as here. Once the ice is gone, the rate of warming *will* dramatically accelerate, followed by horrific ecological and environmental consequences.

Civilization will change in profound ways within a few decades. We will either transition to living more sustainably, or we will destroy ourselves (e.g., Quinn 2002).



Conclusions

Global warming is a challenging topic to teach in part because it is so politicized, multidisciplinary, abstract, and complex. Among misinformation and obfuscation, we tend to cling to our prior beliefs. This classroom demonstration of the temperature kinetics of melting ice provides a compelling example of how scientific apparatus and methods reveal natural truths and our deep misconceptions. The predictive power of theoretical models such as specific and latent heat become evident, and lead to logical generalizations revealing profound consequences of global warming. Although we developed an automated apparatus to measure temperature kinetics, we question if removing the human from the apparatus might make the results more abstract and less credible.

60 Hz noise is a common source of error in electronic measurements. Analog (or digital) notch filters are common methods to attenuate 60 Hz, but we have demonstrated two computationally simple alternatives implementable on an Arduino. Averaging samples uniformly spread over integral 60 Hz periods eliminates the 60 Hz signal, and adding random phase shifts does almost as well and doesn't require measuring precise timing of the sampling.

References and Acknowledgements

- Jaime Ferns collected the melting ice kinetics data in Univ 349. Students in Phys 310 and Chem/Phys 334 commented on the experiment.
- <https://github.com/adafruit/MAX6675-library>
- Hansen J et al. (2016) "Ice melt, sea level rise and superstorms: evidence from paleoclimate data, climate modeling, and modern observations that 2°C global warming could be dangerous". Atmos. Chem. Phys., 16, 3761-3812. <http://www.atmos-chem-phys.net/16/3761/2016/acp-16-3761-2016.html>
- Quinn D (2002) "The New Renaissance". http://www.ishmael.org/Education/Writings/The_New_Renaissance.shtml
- Rasnow B (2015) "Modeling Climate Change in a Test Tube". American Physical Society Annual Meeting. <http://meetings.aps.org/Meeting/FWS15/Session/F4.2>